DECSAI
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Networks

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Networks

- Network Analysis
  - Applications
  - Network Properties
- Network Models
  - Random-Graph Models
  - Growing Random Models
  - Strategic Network Formation
- Network Structure & Dynamics
  - Diffusion through Networks
  - Search on Networks
  - Social Influence Models
  - Networked Markets
- Bibliography

Network Analysis

Networks permeate our lives.

Networks play a central role in determining

- the transmission of information about job opportunities,
- how diseases spread,
- which products we buy,
- our likelihood of succeeding professionally,
- …

Network Analysis

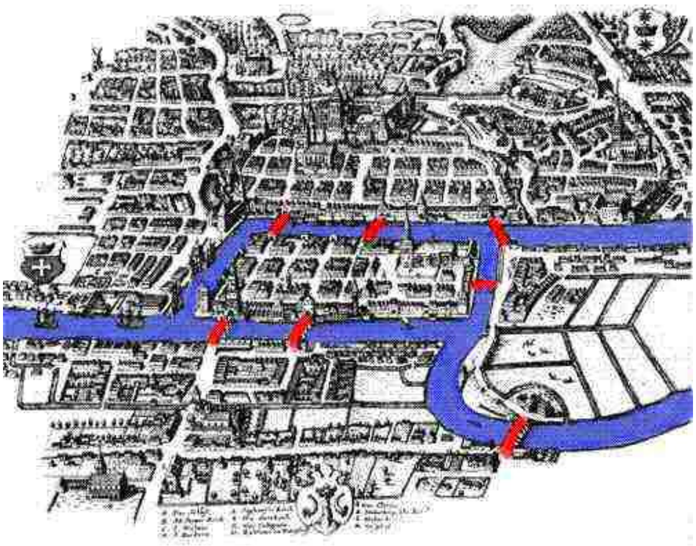
As a field of study…

- How relationships between parts give rise to the collective behaviors of a system and how the system interacts and forms relationships with its environment (complex systems).

- Common principles, algorithms and tools that govern network behavior (network science).

Network Analysis

Origins: Graph Theory



The Seven Bridges of Könisberg
(Leonhard Euler, 1736)

Network Analysis

Networks as graphs "on steroids"…

- **Objects**: Graph vertices.
  - Objects can be of different kinds.
  - Objects can be labeled.
  - Objects can have attributes
- **Links** between objects: Graph edges.
  - Links can be of different kinds.
  - Links can be directed (arcs) or undirected (edges).
  - Links can have attributes.

Network Analysis

A formal definition of network
[Ted G. Lewis: "Network Science," 2009]

G(t) = { N(t), L(t), f(t) : J(t) }

where t = time (simulated or real)
N = nodes (a.k.a. vertices or "actors")
L = links (a.k.a. edges)
f = topology (connections through links)
J = behavior of nodes and links (algorithm)

Network Analysis

An interdisciplinary field: Complex systems
("networks of heterogeneous components that interact")

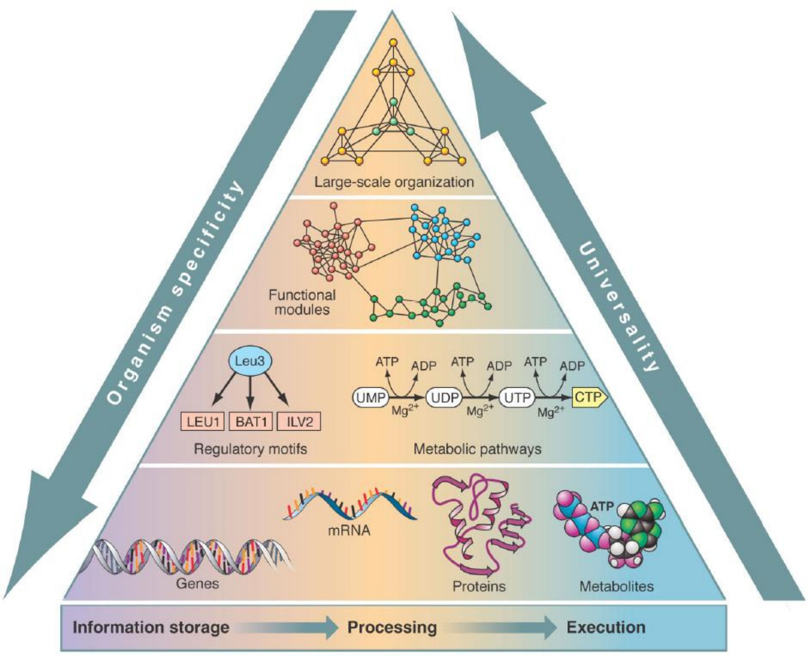
- Physics: **Nonlinear dynamics & chaos**.
  Dynamical systems that are highly sensitive to initial conditions (a.k.a. butterfly effect).
- Economics: **Markets**.
  Spontaneous (or emergent) order as the result of human action, but not the execution of any human design [Austrian perspective].
- Information theory: **Complex adaptive systems**.
  (focus on the ability to change and learn from experience).

Applications

- "Cheminformatics": Chemical compounds.
- "Bioinformatics": Protein networks & bio-pathways
- Software Engineering: Program analysis…
- Network flow analysis (transport, workflows…)
- Semi-structured databases, e.g. XML
- Knowledge management: Ontologies & semantic nets
- Computer-aided design (CAD): IC design…
- Geographic information systems (GIS) & cartography
- Social networks, e.g. Web
- Economic networks, e.g. markets

Applications

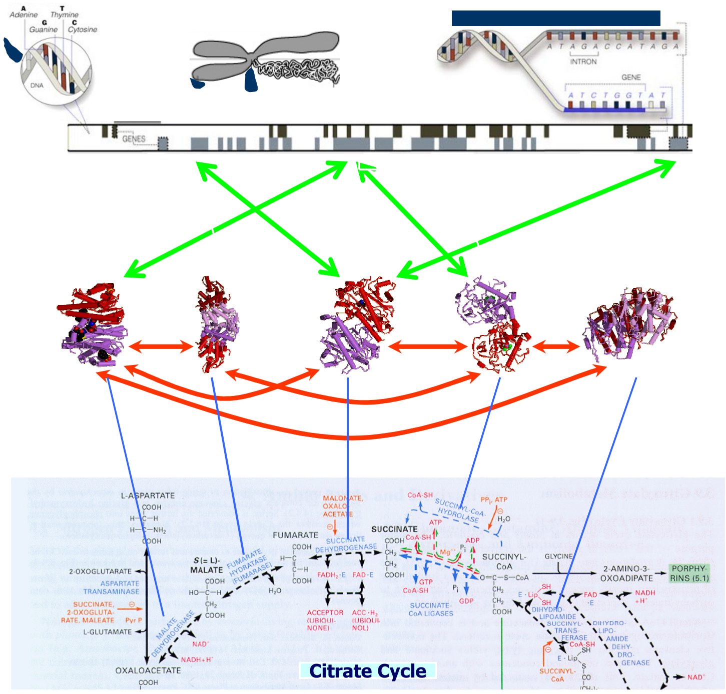
"Life complexity pyramid"



from Z.N. Oltvai and A.-L. Barabasi. Science, 2002

Applications

Biological networks



GENOME

Gene-protein interactions

PROTEOME

Protein-protein interactions

METABOLISM

Biochemical reactions

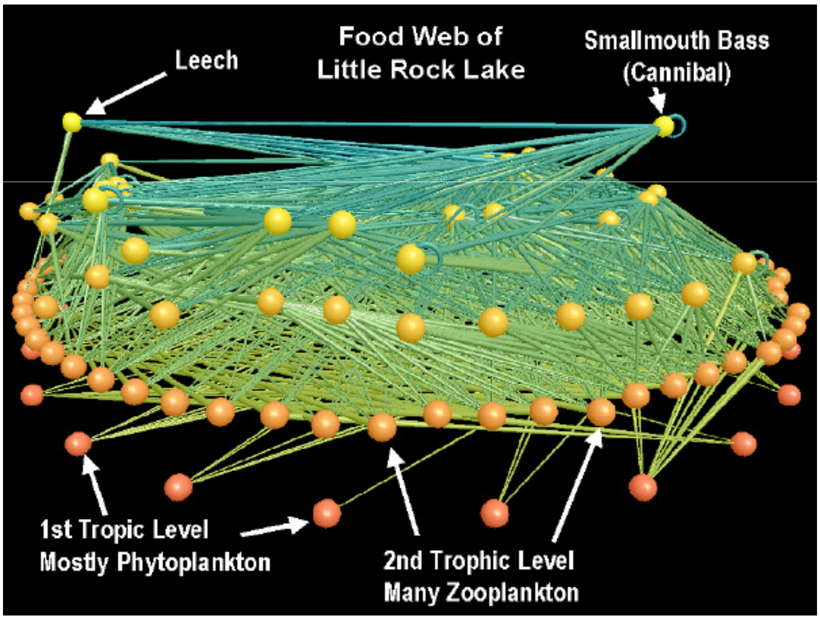
Applications

Yeast protein interaction network

from H. Jeong et al Nature 411, 41 (2001)

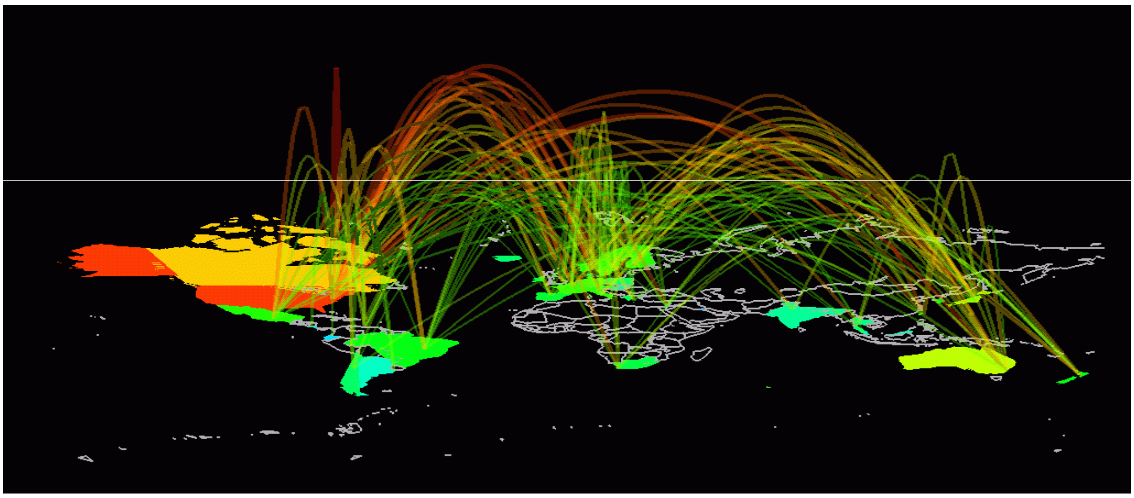
Applications

Ecological network: Trophic relationships in a food web.



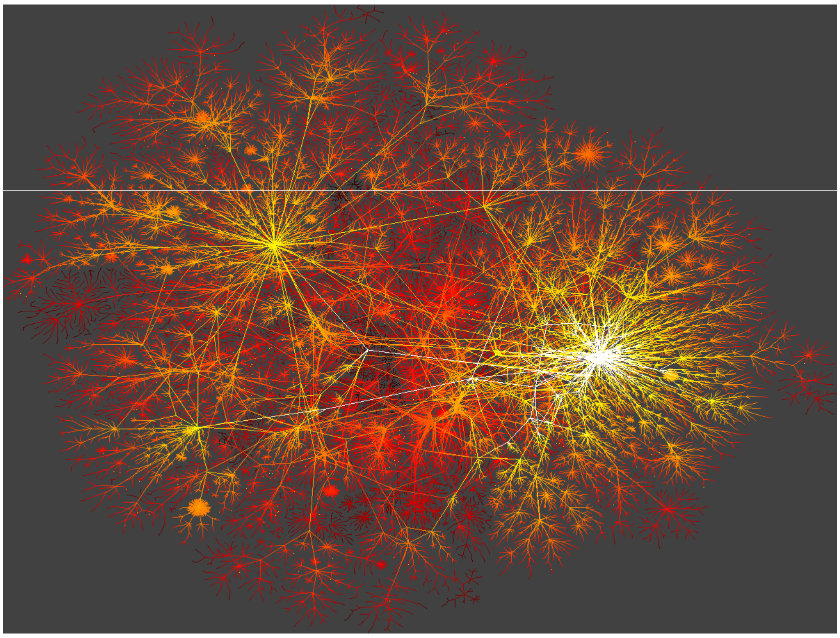
Applications

Telecommunication network



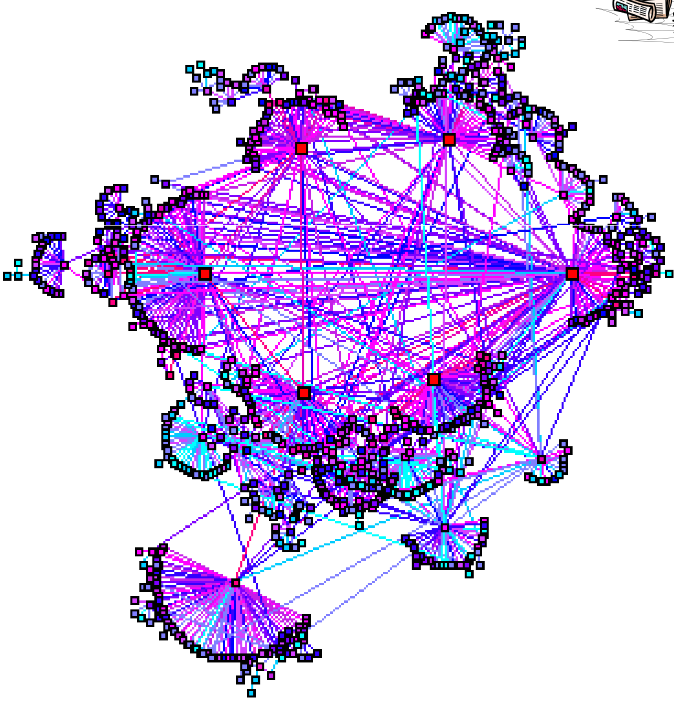
Applications

Internet



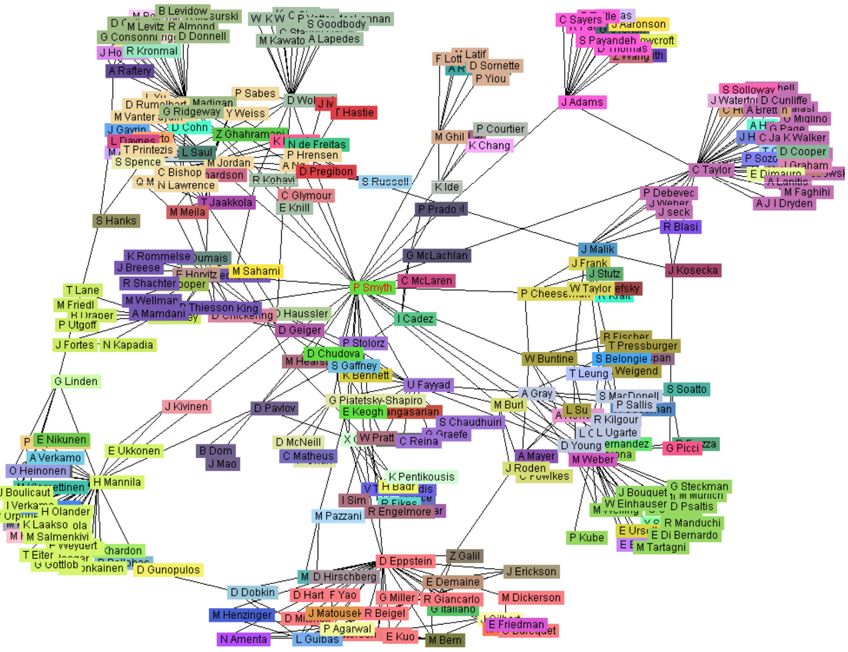
Applications

World Wide Web



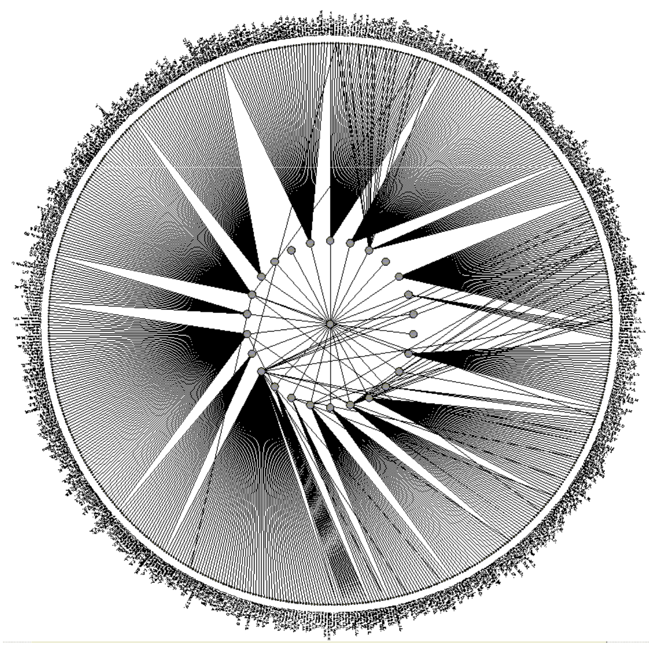
Applications

Social network: Bibliographic network (coauthors)



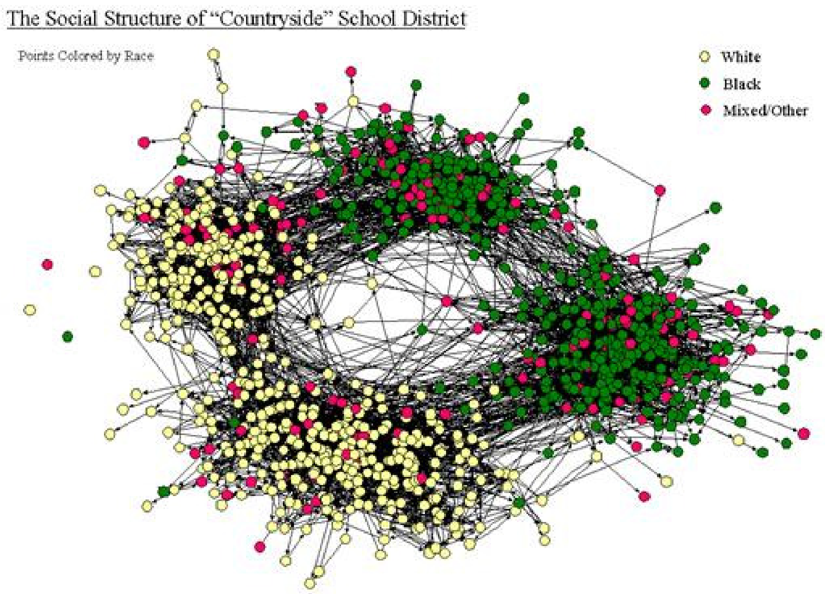
Applications

Social network: Bibliographic network (coauthors)



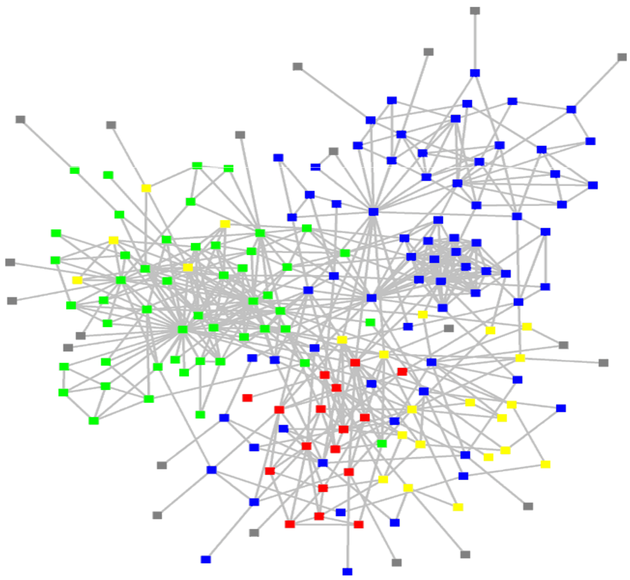
Applications

Social network: FOAF ("friend of a friend")



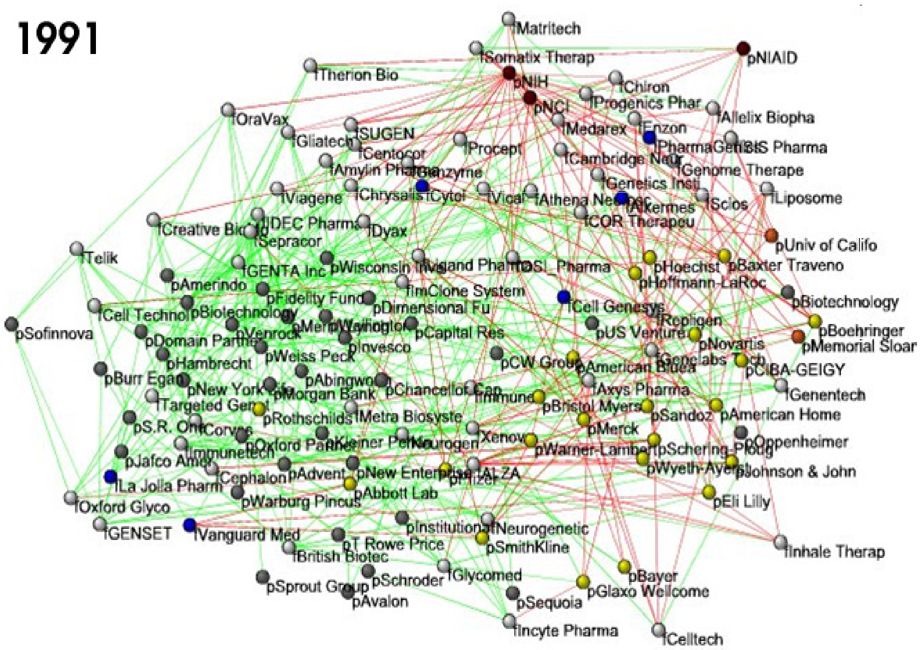
Applications

Social network: Organization



Applications

Social network: US Biotech Industry



Network Properties

Common network features:

- Large scale.
- Continuous evolution.
- Distribution (nodes decide their connections).
- Interactions only through existing links.

Network Properties

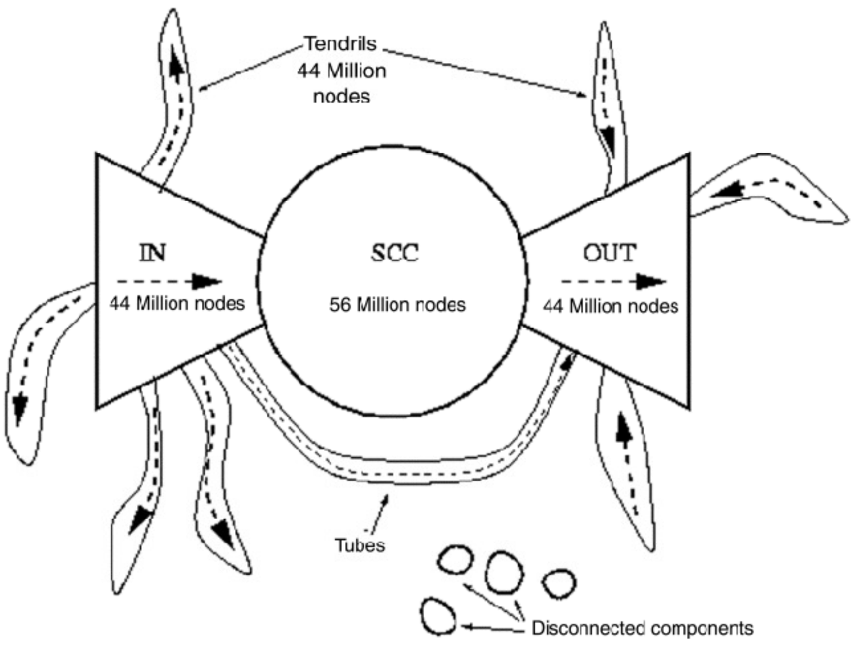
Some interesting structural properties:

- Connected components: How many? Of what size?.
- Network diameter: Average distance, worst case…
- Node degree distribution
  & existence of "hubs" (heavily-connected nodes).
- Groupings (balance between local and large-distance connections, as well as their roles).

Network Properties

Network Connectivity

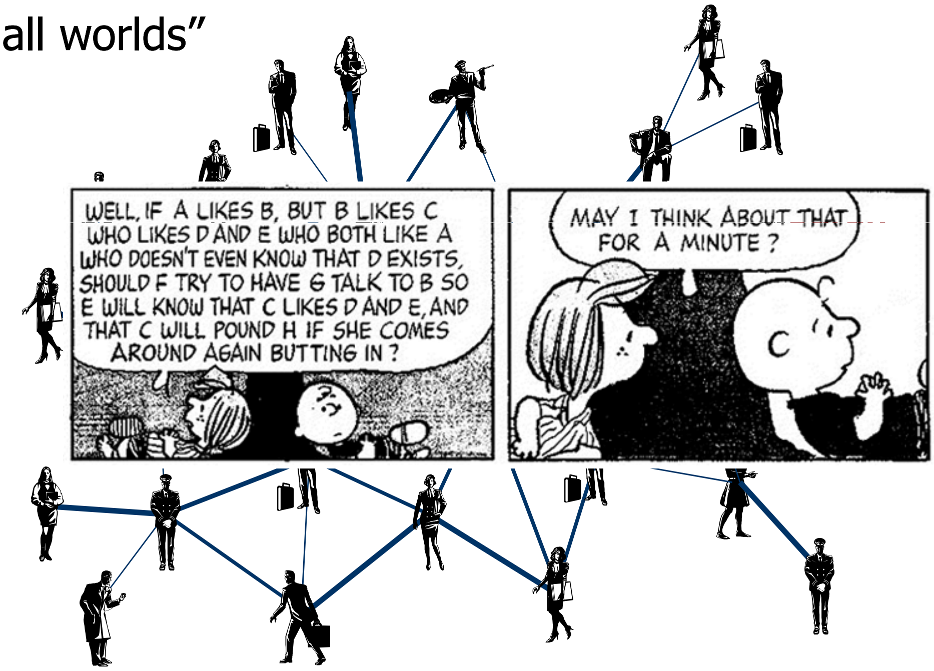
WWW



Network Properties

Network Diameter

"small worlds"



Network Properties

Clustering coefficient

nbr(u)	Neighbors of the node u in the network.
k	Number of neighbors of u, i.e. |nbr(u)|.
max(u)	Maximum number of links among the neighbors of u, e.g. k*(k-1)/2.

Clustering coefficient for the node u:

c(u) = (#links among neighbors of u) / max(u)

Clustering coefficient for the graph G:

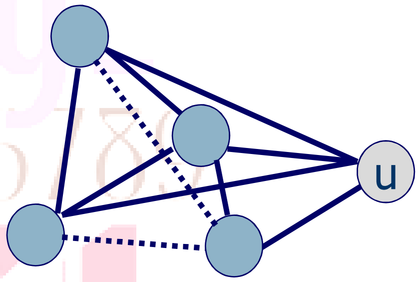
C = average of c(u) for every node in G

Network Properties

Clustering coefficient

k = 4
m = 6

c(u) = 4/6 = 0.66



0 <= c(u) <= 1

Similarity of u neighbors to a clique (complete graph).

Informal interpretation:
"My friends tend to be friends among them."

Network Properties

Clustering coefficient for some real networks

Network	N	C	C_{rand}	L
WWW	153127	0.1078	0.00023	3.1
Internet	3015-6209	0.18-0.30	0.001	3.7-3.76
Actor	225226	0.79	0.00027	3.65
Coauthorship	52909	0.43	0.00018	5.9
Metabolic	282	0.32	0.026	2.9
Foodweb	134	0.22	0.06	2.43
C. elegance	282	0.28	0.05	2.65

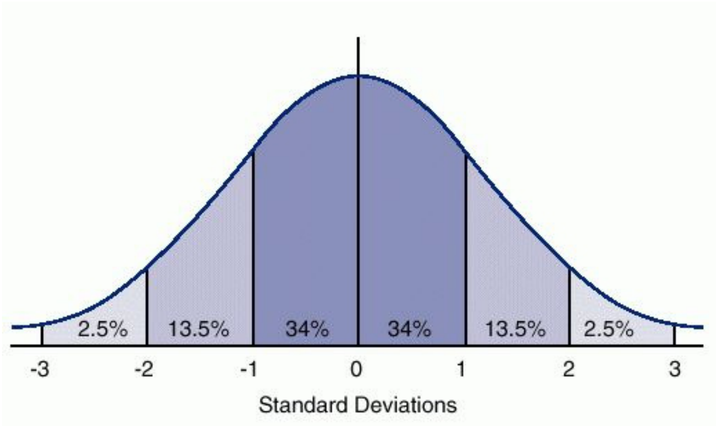
Clustering coefficient (C): $C>C_{rand}$
Path length (L): $L<L_{rand}$

Network Properties

Node degree distribution

Normal distribution

Parameters: Average & deviation

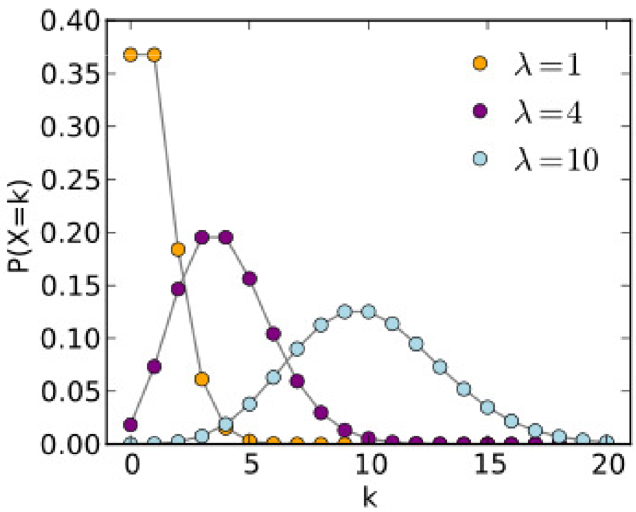


Network Properties

Node degree distribution

Poisson distribution

Single parameter: λ (mean & deviation)

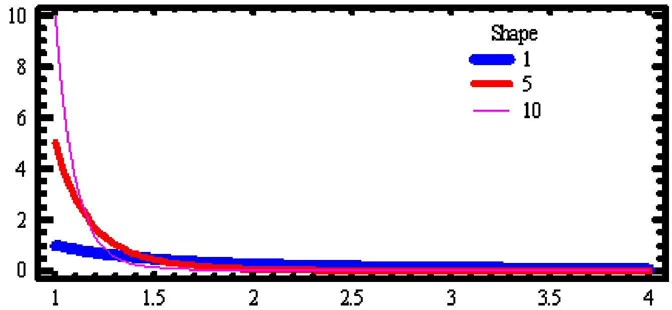


Network Properties

Node degree distribution

Pareto distribution (a.k.a. "power law")

Single parameter: α



$$P(x) \sim x^{-\alpha}$$

The Pareto principle (the "80-20 rule"):

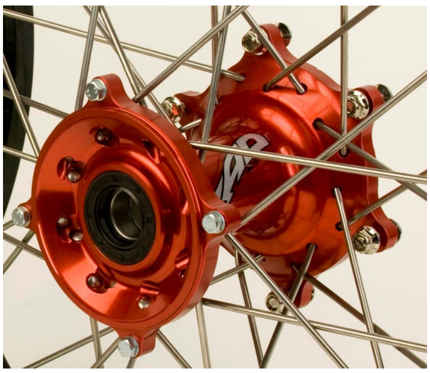
20% of the population controls 80% of the wealth.

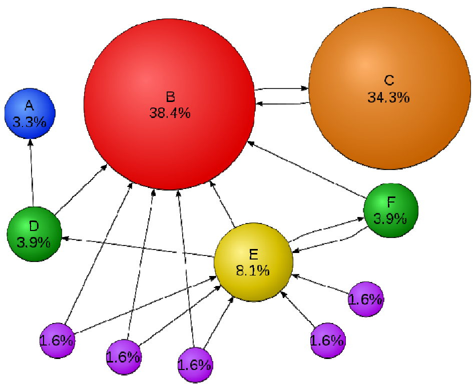
Network Properties

Node degree distribution

Hubs

Small number of nodes with a very high degree.





- Hubs appear with power laws (**P(x)** $\sim x^{-\alpha}$), but not with normal/binomial/Poisson distributions.

Network Properties

Node degree distribution

Log-log plot

- **Pareto distribution**
  - $\log(\Pr[X = x]) = \log(1/x^\alpha) = -\alpha \log(x)$
  - Linear, $-\alpha$ slope.
- **Normal distribution**
  - $\log(\Pr[X = x]) = \log(a \exp(-x^2/b)) = \log(a) - x^2/b$
  - Nonlinear, concave around the average.
- **Poisson distribution**
  - $\log(\Pr[X = x]) = \log(\exp(-\lambda)\ \lambda^x/x!)$
  - Nonlinear.

Network Properties

Node degree distribution

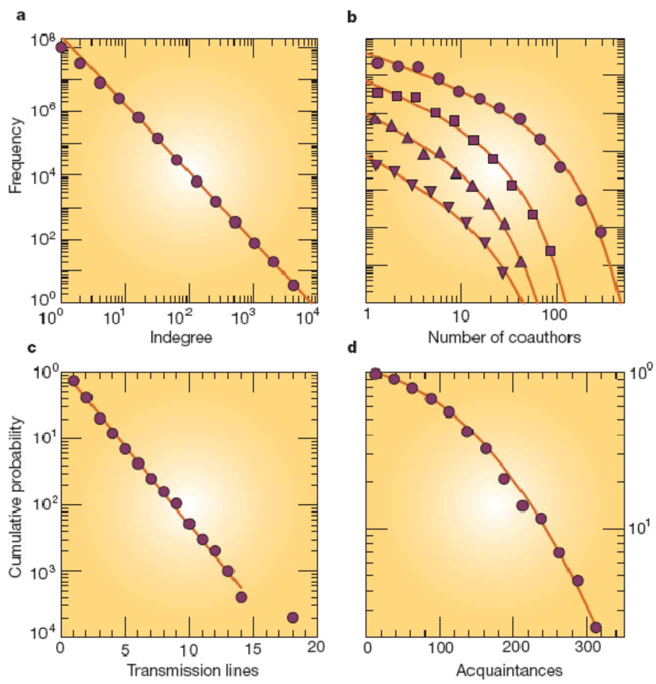
Log-log plot

a WWW
power law

b Coauthorship networks
power law with exponential cutoff

c Power grid
exponential

d Social network
Gaussian



Network Models

"Natural" networks tend to have…

- One (or a few) connected components.
  - Independent of network size.
- A small diameter ("six degrees of separation").
  - Constant, logarithmically increasing, or even decreasing with network size.
- High clustering ("communities").
  - Much larger than expected from a random network (and, even so, with a small diameter!).
- A mixture of connections.
  - Local vs. "long-distance" connections

Do they share some "universal" features?

Network Models

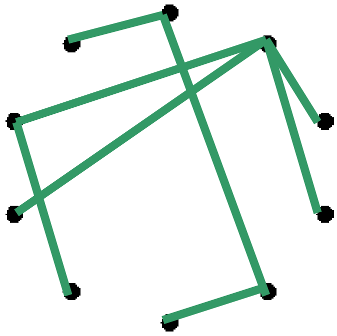
- Random networks.
- Random-biased networks.
- Small-world networks.
- Scale-free networks.
- Hierarchical & modular networks.
- Affiliation networks.

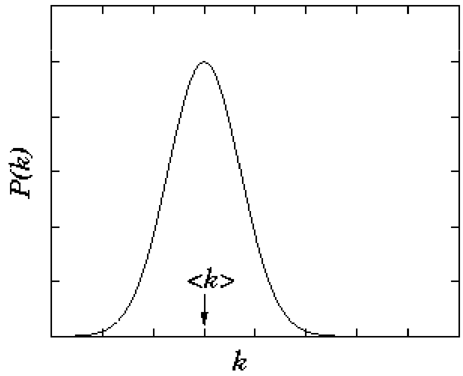
Network Models

Random Networks

Erdös-Rényi model

- Small number of connected components (typically one).
- Low clustering coefficient.
- Poisson distribution.

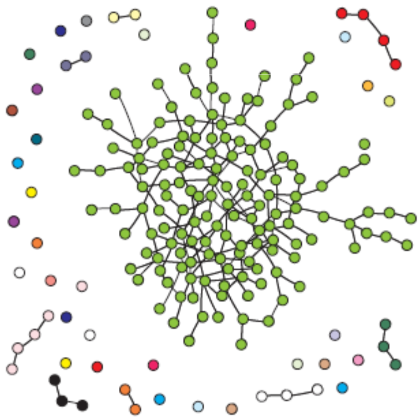


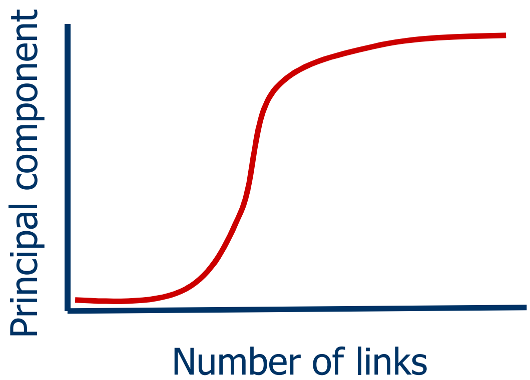


Network Models

Random Networks

Erdös-Renyi model

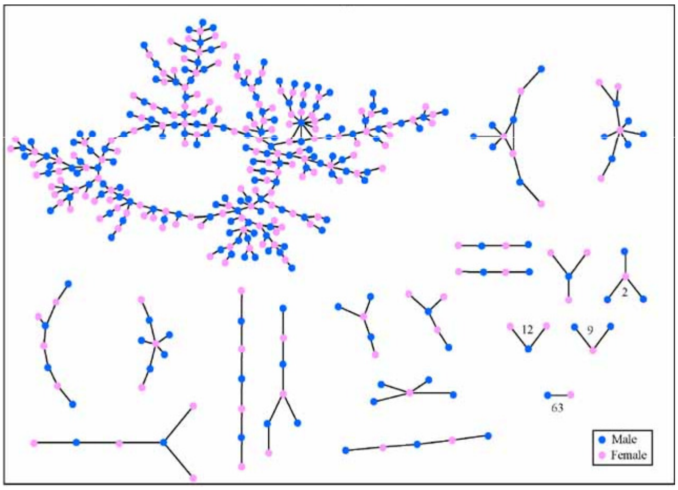


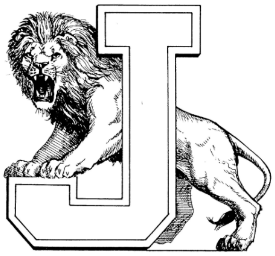


Network Models

Random Networks

Example: Romantic relationships in the Add Health data set.





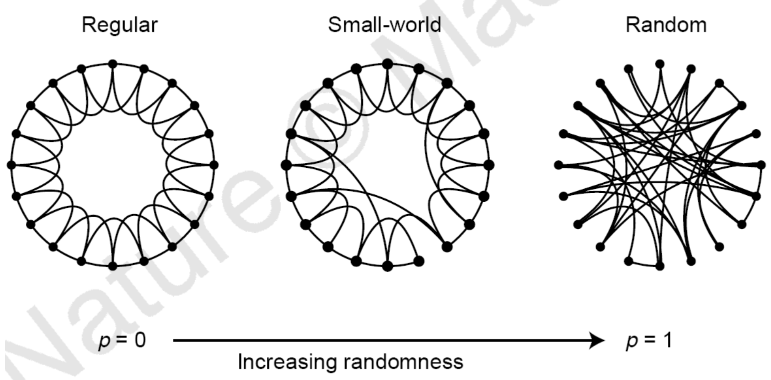
Peter S. Bearman, James Moody & Katherine Stovel:
"Chains of Affection: The Structure of Adolescent Romantic and Sexual Networks"
American Journal of Sociology, 110(1):44–91, July 2004

Network Models

Small-World Networks

Watts & Strogatz model

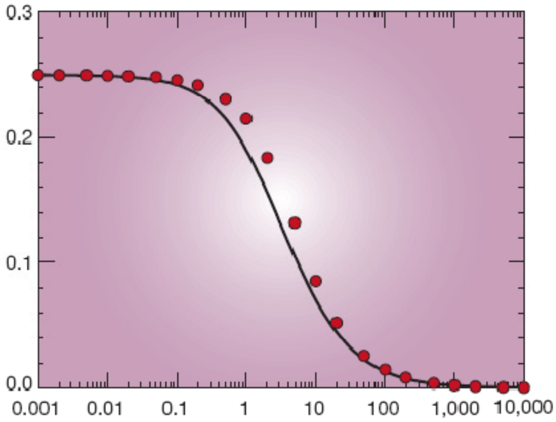
- Small number of connected components (typically one).
- **Small diameter.**
- Poisson distribution.
- **High clustering coefficient.**



Network Models

Small-World Networks

Watts & Strogatz model



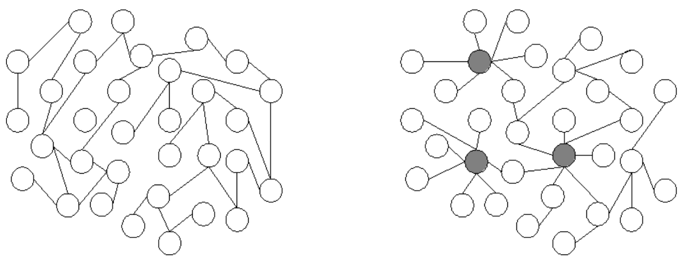
Average path length, normalized by system size,
plotted as a function of the average number of shortcuts.

Network Models

Scale-Free Networks

Barabási & Albert model

- Small number of connected components (typically one).
- Small diameter.
- **Pareto distribution.**
- **Small clustering coefficient.**
- **Hubs.**



(a) Random network (b) Scale-free network

Network Models

Scale-Free Networks

Barabási & Albert model

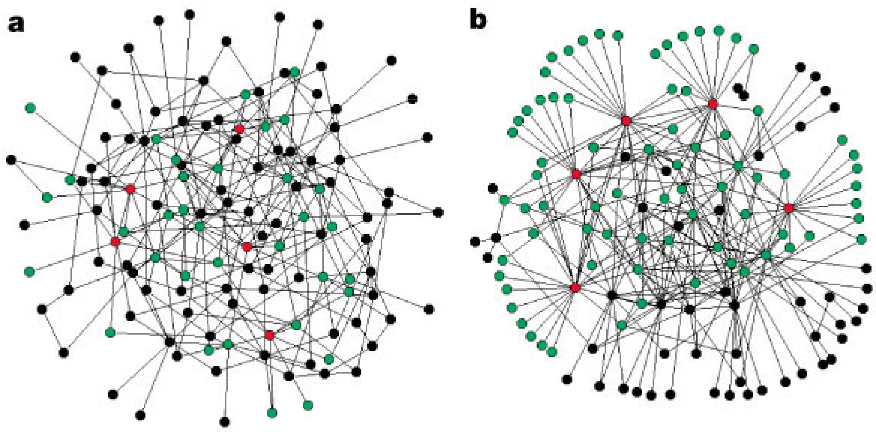
"Natural" interpretation of the model:

- Variable number of nodes:
  Network grows as new nodes are added.
- **Preferential attachment**:
  The more connected a node is,
  the more likely it is to receive new links
  ("rich get richer" or Matthew effect).

Network Models

Scale-Free Networks

Barabási & Albert model

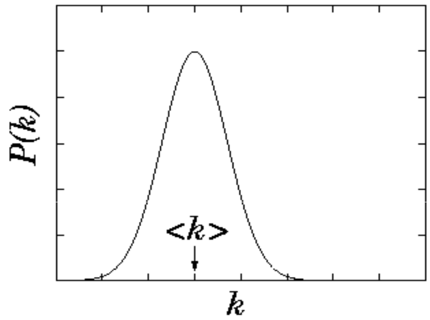


Exponential model…
… without hubs.

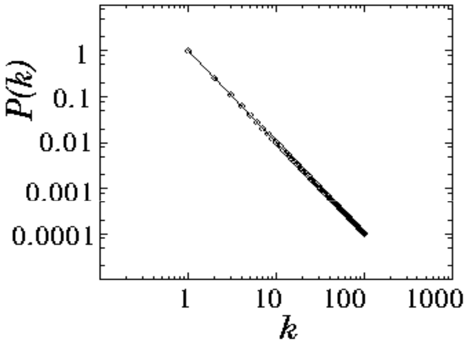
Scale-free model…
… with hubs.

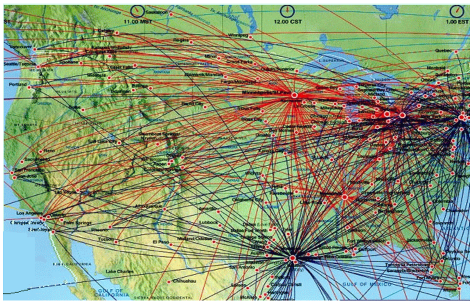
Network Models

Poisson



Pareto (power law)

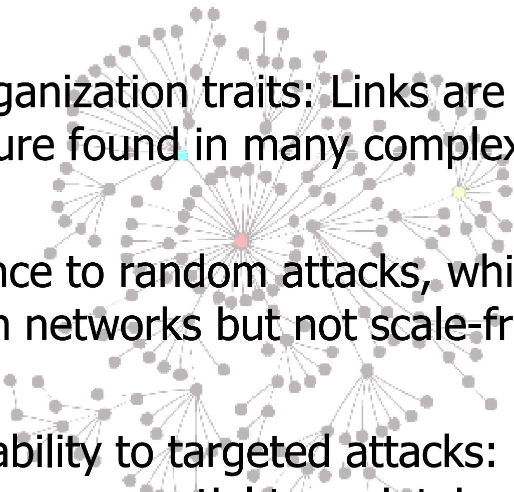




Network Models

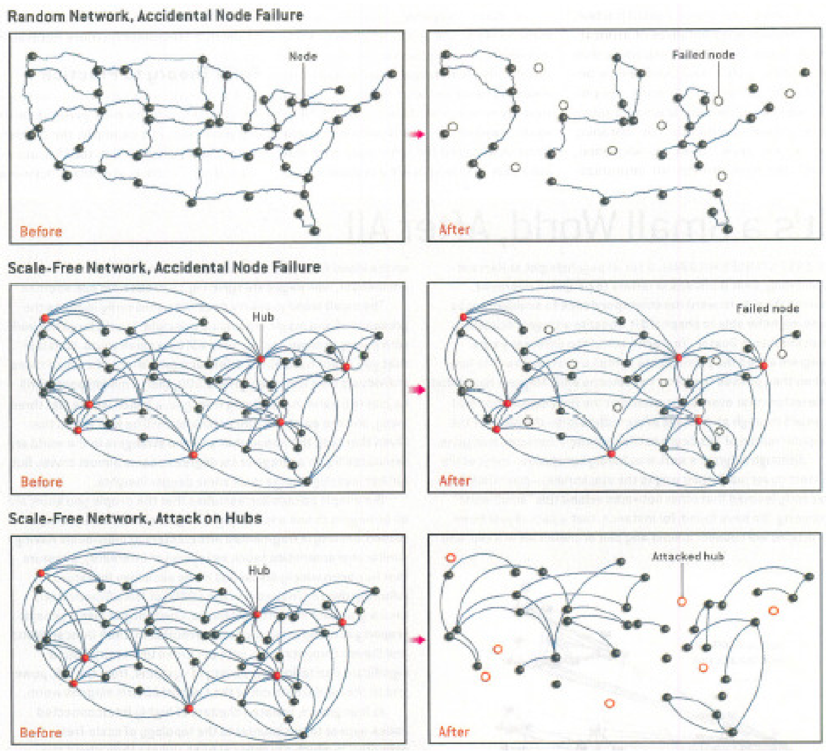
Scale-Free Networks

Features



- Self-organization traits: Links are not random (a feature found in many complex systems).
- Tolerance to random attacks, which easily disrupt random networks but not scale-free networks.
- Vulnerability to targeted attacks: "Hubs" are essential to maintain connectedness.

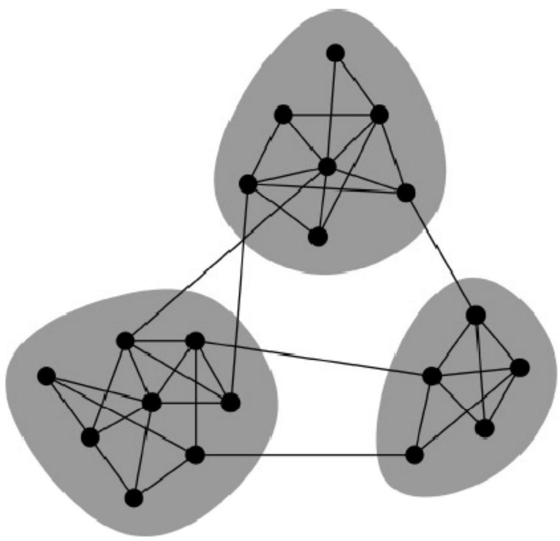
Network Models



Network Models

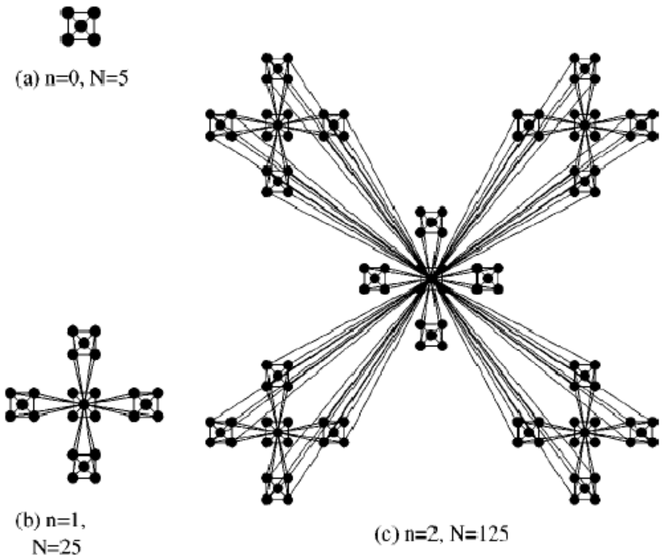
Hierarchical/Modular Networks

- Hierarchical organization.
- Hubs.
- **Cliques**.



Network Models

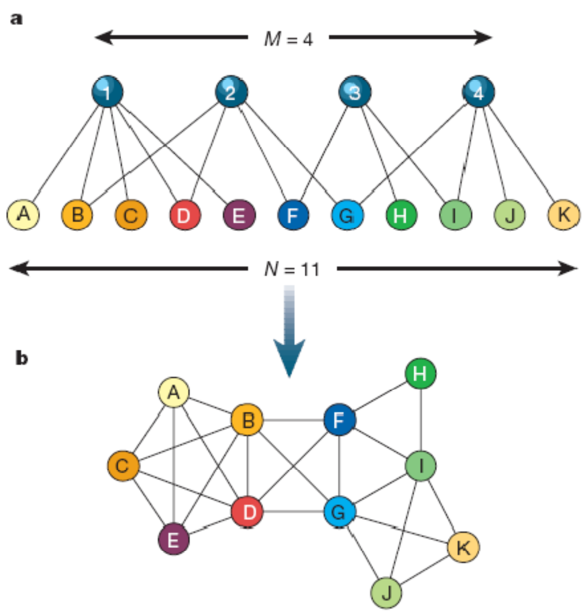
Hierarchical/Modular Networks



Network Models

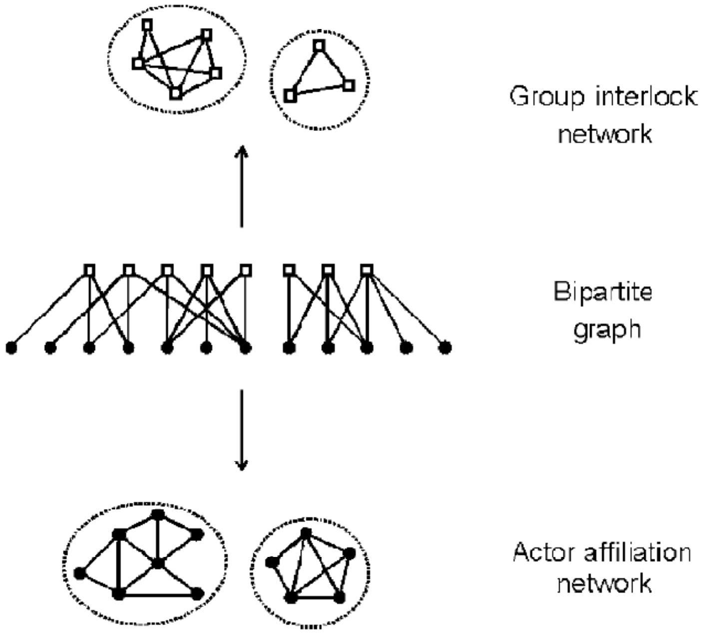
Affiliation Networks

Bipartite graph to model social interactions:

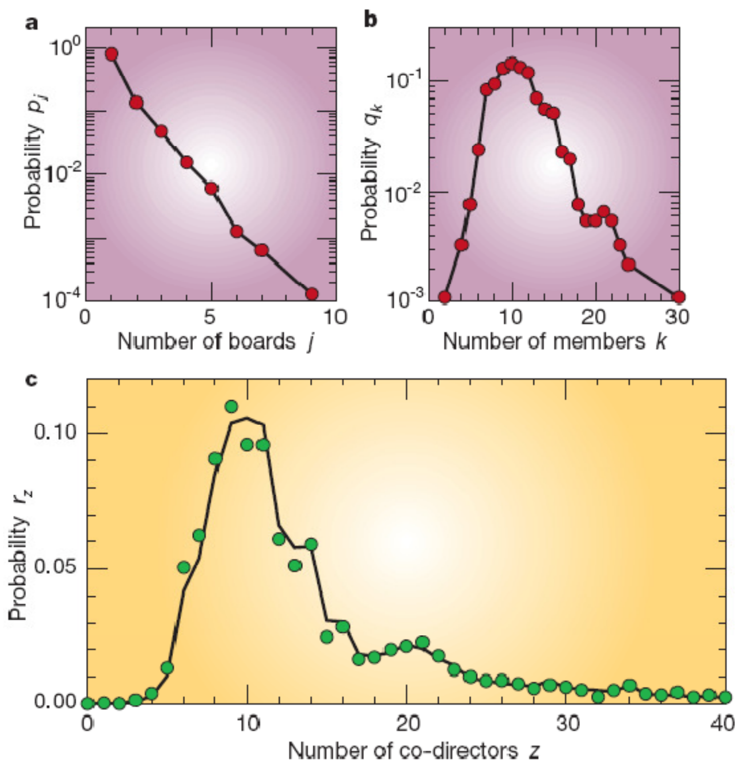


Network Models

Affiliation Networks



Network Models



Network Structure & Dynamics

The countless ways in which network structures affect our lives make it critical to understand:

1. How network structures affect behavior.
2. Which network structures are likely to emerge.

Network Structure & Dynamics

A complex system is a system composed of interconnected parts that, as a whole, exhibit one or more properties (behavior) not obvious from the properties of the individual parts (i.e. emergence).

Network Structure & Dynamics

Research problems

- Search on networks
  (with partial local information)
- Diffusion problems:
  epidemics, social contagion (ideas, fads, products…)
- Analysis of network properties
  e.g. robustness/vulnerability

Network Structure & Dynamics

From an algorithmic point of view…

- Objects:
  - Ranking (HITS, PageRank…).
  - Classification & anomaly detection.
  - Clustering & community analysis.
  - Object identification (e.g. "entity resolution").
- Links:
  - Link prediction.
- Graphs:
  - Subgraph detection.
  - Graph classification.
  - Graph generation models.
  - …

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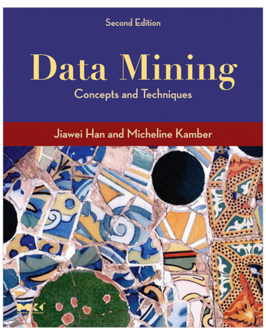

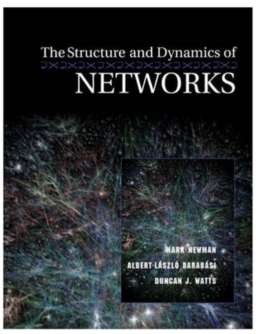

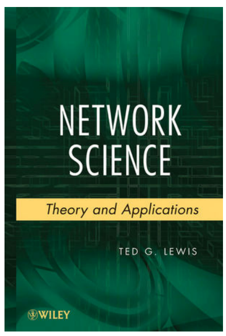

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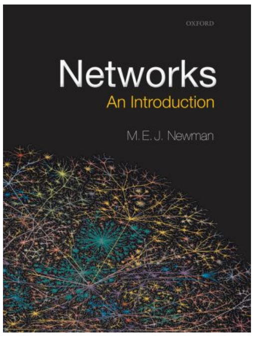

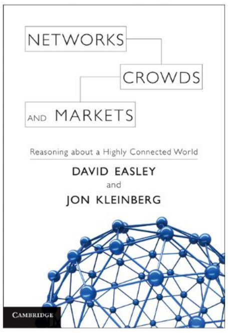

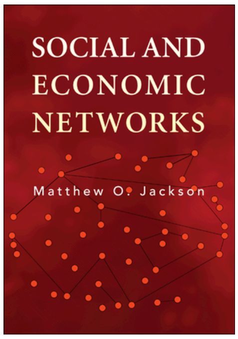

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